Photon-energy dissipation caused by an external electric circuit in "virtual" photo-excitation processes

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We consider generation of an electrical pulse by an optical pulse in the "virtual excitation" regime. The electronic system, which is any electro-optic material including a quantum well structure biased by a dc electric field, is assumed to be coupled to an external circuit. It is found that the photon *frequency* is subject to an extra red shift in addition to the usual self-phase modulation, whereas the photon *number* is conserved. The Joule energy consumed in the external circuit is supplied only from the extra red shift.

42.65.Vh, 78.66.-W, 03.65.Bz

Virtual excitation of electronic systems by optical fields has been attracting much attention recently [1–5]. Here, "virtual" means roughly that the photon energy is lower than the absorption edge by some detuning energy Δ, so that photon absorption does not take place. More precisely, it means that the excitation takes place *adiabatically* so that no real transitions occur and the quantum-mechanical coherence is preserved. For a simple two-level system (with a long dephasing time), for example, the excitation will be "virtual" if

$$(T_{\rm tr}/\hbar)^2 \gg (\mu\mathcal{E}/\Delta^2)^2, \tag{1}$$

where $T_{\rm tr}$ is the transient time for which the envelope of the optical pulse changes appreciably, μ denotes the transition dipole moment of the two-level system, and $\mathcal{E}$ is the envelope of the electric-field amplitude of the optical pulse. The concept of the virtual excitation has been widely used, for example, to describe ultrafast nonlinear optical responses [1–5]. It was also shown that a 'quantum non-demolition (QND)' measurement of the photon number N (*i.e.*, measurement of N without changing its statistical distribution) is possible by the use of the virtual excitation of an electron interferometer [6].

On the other hand, many studies have recently been devoted to generation of an electrical pulse by excitation of a material by a short optical pulse [2–5,7]. For the material, we can basically use any materials which possess finite electro-optic (EO) coefficient, $\chi^{(2)} \equiv \chi^{(2)}(0;-\omega,\omega)$. Of particular interest is a quantum well structure (QWS) biased by a dc electric field [2,3]. The dc field is applied to induce large $\chi^{(2)}$, which results in high efficiency for the generation of an electrical pulse. In particular, it was suggested that the ultrafast response would be obtained by working in the "virtual excitation" regime [2,3]. However, present understanding seems quite insufficient to investigate such a fancy combination of the idea of the electrical-pulse generation with the concept of the virtual excitation.

In this paper, we raise and answer the following fundamental questions on the electrical-pulse generation by virtual photo-excitation: (i) What is the state of the optical pulse after it passes through the EO material? In particular, what is the photon energy and photon number? (ii) What role is played by the external electric circuit in determining the photon state? (iii) When the material is a biased QWS, what supplies the energy to the electrical pulse — an external battery (which induce the dc bias field) or the optical field? (iv) Is it possible to perform a QND measurement by monitoring the generated electrical pulse?

Let us start with a biased QWS. We suppose that metallic contacts are deposited on both sides of a QWS sample in order to apply the static bias field F_0 (>0) by an external battery V_0 (Fig. 1). The sample thus works as a capacitor, whose capacitance is $\epsilon L \equiv C_0$, where ϵ denotes the linear dielectric constant at low frequencies. Exciton states of the QWS are strongly deformed by F_0, and each exciton acquires a large *static* dipole moment, l [2,3].

It is convenient to describe the exciton dynamics *in terms of such deformed states*. If, for simplicity, we look at the lowest-exciton state only, an effective Hamiltonian in the optical field, $\mathcal{E}\cos\omega t$, may then be written as [8]

$$H = \varepsilon_x a^\dagger a - \mu(a^\dagger + a)\mathcal{E}\cos\omega t - l(F_P + F_1)a^\dagger a \tag{2}$$

where $a^\dagger$ (a) and ε_x denote the creation (annihilation) operator and the energy, respectively, of the deformed exciton state. When the detuning energy $\Delta \equiv \varepsilon_x - \hbar\omega$ satisfies Eq. (1), the optical field excite the deformed excitons virtually, which induce the static electric field $F_P = -l\langle a^\dagger a\rangle/\epsilon_0$ [9], which is nonzero (<0) only in the well region [2,3]. Since l is large ($=10^{1-2}$ e Å), $|F_P|$ becomes large, which results in a large EO coefficient $\chi^{(2)}$ [2–5]. On the other hand, to cancel out F_P, current J is induced which alters the surface charge density of the metallic contacts from the equilibrium value $\sigma_0 \equiv \epsilon F_0$ into $\sigma = \sigma_0 + \sigma_1$, and σ_1 generates the canceling field

$F_1 = \sigma_1/\epsilon$. Therefore, the total dc field in the QWS is $F = F_0 + F_P\Theta(z) + F_1$, where Θ is a unit step function which is nonzero only in the well region. The equation of motion of σ_1 may thus be

$$\frac{d\sigma_1}{dt} = -\frac{\sigma_1}{C_0 R} - \kappa\frac{F_P}{RL}, \tag{3}$$

where $\kappa \equiv$ well thickness$/W$. The models of the previous work [2,3], which assumed the absence of the external circuit, correspond to the limit of $R \to \infty$ of our model.

It is seen that the optical field $\mathcal{E}$ interacts with the quantum-mechanical excitonic variables a, $a^\dagger$ via the μ term in Eq. (2), and the excitonic variables interact with the classical surface charge σ_1 via the F_P and F_1 terms in Eqs. (2) and (3). The interesting point here is that *only* the motion of σ_1 suffers explicit dissipation; the C_0R term in Eq. (3). We will show below that this dissipation eventually causes, through a chain of interactions, energy dissipation in the optical field.

We first note that F_0 has not appeared explicitly in Eqs. (2) and (3): all effects of F_0 have been incorporated only in the deformed exciton state which defines a, $a^\dagger$, μ, and l. Consequently, the external battery which produces F_0 supplies *no* net energy: the Joule energy RJ^2 must be supplied by something else — the only possible supplier is the optical field. The role of the battery (and F_0) is just to produce large $\chi^{(2)}$. The absence of energy supply from the battery will be confirmed also in the following calculations.

We are interested in the evolution of the optical field and energy flow. We here evaluate them to $O(\mathcal{E}^2)$, because the analysis [10] which includes the third-order nonlinear effects shows that the second-order effects are essential [11]. We also found [10] that concerning the quantities which we will discuss below the microscopic model of Eq. (2) gives the same results as a phenomenological model in which the excitonic (or electronic) system is phenomenologically treated as a transparent EO material. The only difference is that in the former model $\chi^{(2)}$ is obtained by solving Eq. (2), whereas in the latter $\chi^{(2)}$ is a given parameter. The phenomenological model is therefore applicable to any transparent EO materials including the biased QWS. For this reason, we hereafter present our results in the language of the phenomenological model: for example, F_P is now

$$F_P = -(\epsilon_0/\epsilon)\chi^{(2)}\mathcal{E}^2, \tag{4}$$

where $\chi^{(2)}$ is, as in the case of the biased QWS, the value of $\chi^{(2)}$ *in the presence of* F_0.

To avoid inessential complexities, we assume that the light intensity is almost constant over the cross-section of the optical beam, and also that the cross-section agrees with that ($W \times W$) of the capacitor. In the propagating direction x, the optical pulse is assumed to have a portion (of length cT) of constant intensity inbetween the initial and final transient portions of length $cT_{\rm tr}$. To focus on new phenomena only, we assume that $T_{\rm tr} \ll C_0R,\ T$; under this condition σ does not change during the transients and thus what happens in the optical field of the transient portions is just the usual self-phase modulation, which is well-known and of no interest here. We therefore focus on the constant-intensity portion, and take $t = 0$ as the time at which that portion begins to enter the capacitor. We further assume, for simplicity, that $L \ll cT/n$ (*i.e.*, $\mathcal{E} \approx$ constant in the capacitor), where L is the length of the capacitor, c the light velocity in vacuum, and n the refractive index.

Under these conditions, Eq. (3) can be easily solved to give

$$\sigma_1 = \begin{cases} \kappa\epsilon|F_P|(1 - e^{\frac{-t}{C_0R}}), & (0 \le t \le T) \\ \kappa\epsilon|F_P|(1 - e^{\frac{-T}{C_0R}})e^{\frac{T-t}{C_0R}}, & (T < t) \end{cases} \tag{5}$$

where F_P is given by Eq. (4), and κ is now $\kappa \equiv$ thickness of the EO material$/W$. Associated with the time-varying σ_1 is the current $J = \frac{\partial}{\partial t}WL\sigma_1$, which generates the Joule heat in the resistance R [12];

$$\begin{aligned} U_R &= \int_{-\infty}^{\infty} RJ^2 dt \\ &= (\kappa WL\epsilon_0\chi^{(2)}\mathcal{E}^2)^2(1 - e^{-T/C_0R})/C_0. \end{aligned} \tag{6}$$

Let us find out the supplier of this energy — the battery or the optical field? The work done by the battery is

$$U_{V_0} = \int_{-\infty}^{\infty} V_0 J dt = V_0 WL[\sigma_1(\infty) - \sigma_1(0)], \tag{7}$$

which is zero because, as seen from Eq. (5), $\sigma_1(\infty) = \sigma_1(0) = 0$. That is, the battery does not supply net energy at all, in agreement with the observation we have drawn above from the microscopic model. Therefore, the only possible supplier of the Joule energy is the optical field, the evolution of which we will investigate now.

We note that the dc field F in the EO material varies from F_0 (for $t < -T_{\rm tr}$) to $F_0 + F_P + F_1$ (for $0 \le t \le T$) and then to $F_0 + F_1$ (for $T + T_{\rm tr} < t$), where $F_1 = \sigma_1/\epsilon$ also varies according to Eq. (5). The time-dependent F produces the time-dependent change of the refractive index n:

$$\delta n = \chi^{(2)}(\omega;\omega,0)(F - F_0)/2n, \tag{8}$$

where n and $\chi^{(2)}(\omega;\omega,0)$ denote their values *in the presence of* F_0. With the help of the symmetric relation, $\chi^{(2)}(\omega;\omega,0) = 4\chi^{(2)}(0;-\omega,\omega)$, Eqs. (5) and (8) yield, for $0 \le t \le T$,

$$\delta n = -(2\epsilon_0/n\epsilon)|\chi^{(2)}|^2\mathcal{E}^2[1 - \kappa(1 - e^{-t/C_0R})]. \tag{9}$$

By this time-dependent δn, the optical field undergoes a frequency shift (chirping) [10]. When $L \ll C_0Rc/n$, the shift is simply given by

$$\delta\omega = -\frac{\partial}{\partial t}\frac{\omega L\kappa\delta n}{c}$$
$$= -\frac{2\epsilon_0\omega\kappa^2|\chi^{(2)}|^2\mathcal{E}^2 L}{n\epsilon c C_0 R}e^{-t/C_0R}. \quad (10)$$

Here κ in the first line has appeared because δn is large only in the EO material.

We find that (i) the optical field undergoes a red shift, (ii) the shift approaches zero in both limits of $R \to \infty$ and $R \to 0$, and (iii) the shift becomes maximum at the beginning ($0 \le t \ll C_0R$) of the constant-intensity portion of the optical pulse, and decays exponentially after that.

This shift is a kind of a self phase modulation (SPM) process in the sense that the shift is driven by the optical field *itself*. However, it is totally different from the *usual* SPM, which generally occurs when an optical pulse passes through a nonlinear medium. To distinguish between the two, we hereafter call the shift of Eq. (10) the 'extra shift' or 'extra red shift.' Major differences are; (a) In contrast to Eq. (10), the usual SPM is basically independent of the external circuit — it occurs, for example, even when $R \to \infty$. (b) The *total* energy of the optical pulse is conserved in the usual SPM process (because the frequency shifts occur in the opposite directions at the initial and final transients), whereas the extra shift results in loss of the total energy (Eq. (11) below). (c) The usual SPM is approximately instantaneous (delay $\approx$ response time of nonlinear processes), whereas the extra shift occurs with a considerable delay ($\sim C_0R$) — the shift takes place during $0 \lesssim t \lesssim C_0R$ in order to compensate for F_P which is established at $t = 0$. These differences arise because the extra shift is a property of the *coupled* system of a nonlinear EO material and an external electric circuit, whereas the usual SPM is a property of a nonlinear material only.

In terms of the microscopic model, the physics of the extra shift may be understood as follows: Photons (virtually) excite excitons of energy $\varepsilon_x - l(F_P + F_1)$, and the excited excitons will emit photons subsequently. Here, the energy of the excitons is decreasing as t goes by because $F_1 = \sigma_1/\epsilon$ is increasing according to Eq. (5). As a result, the emitted photons have lower energies than the (virtually) absorbed photons. Hence the red shift, and its magnitude decays exponentially with the same decay constant as that of σ_1.

The magnitude of the extra shift, Eq. (10), depends on many material and structural parameters. For example, for a 100-Åwell/ 100-Åbarrier multiple quantum well structure for which $\kappa \approx 1/2$, the red shift is estimated to be of the order of $10^2 L$ MHz when $I \sim 10^2$ MW/cm^2, $F_0 \sim 10^2$ kV/cm, $T \sim C_0R \sim 1$ ps, $\Delta = 10$ meV, and L here is measured in μm,

Our next task is to find out the energy flow. To do this, we for the moment assume that the photon number is conserved (this assumption will be justified soon). In this case, the loss of the light intensity $I = \epsilon_0 cn\mathcal{E}^2/2$ is given by $\delta I = I\delta\omega/\omega$. Therefore, the loss of the total photon energy due to the extra red shift is

$$U_{ERS} = \int_0^T \left|W^2\delta I\right| dt$$
$$= W^2 IL\kappa[\delta n(0) - \delta n(T)]/c. \quad (11)$$

Inserting Eq. (9) and $C_0 = \epsilon L$, and comparing with Eq. (6), we find that $U_{ERS} = U_R$. Therefore, *all the Joule energy is supplied by the extra red shift of the optical field.* It also shows that *the photon number N is conserved*, because if it were lost then $U_{ERS} < U_R$. This indicates that the present photon-energy dissipation *cannot be described as a simple dephasing process*, which is accompanied by loss of N.

We have thus found that when an optical pulse excites the electronic system of Fig. 1 'virtually' (in the sense that Eq. (1) is satisfied) then the final state of the optical pulse has the *same* number of photons as the initial state. However, the frequencies of photons are lowered, which are consumed to generate the Joule heat in the external circuit. This is in a marked contrast to the virtual photo-excitation of an electron interferometer which was discussed in [6], where it was shown that *both* the number and frequencies of photons are conserved. Since we can estimate the photon number N by measuring the interference currents, the electron interferometer works as a QND photo-detector [6]. We can estimate n also in the present case by, say, monitoring the voltage drop across the resistance. Can we call it a QND measurement? The answer clearly depends on the definition of the QND measurement. Kitagawa [13] proposed to accept it as a QND measurement in a broad sense. To perform a QND measurement in the narrow sense (*i.e.*, both N and frequencies are conserved), one may use the scheme of Ref. [4], in which the generated voltage modulates an electron interference current in an electro-static Aharonov-Bohm interferometer.

We have thus found that when you try to get information on photons through virtual photo-excitation of an electronic system, the photon energy will or will not be conserved depending on the detailed structures of the electronic system *and* the external circuit (although the circuit is *not* directly connected to the optical field). These findings shed light on the theory of measurement of photons using electronic systems.

Finally, let us comment on the case in which the external circuit of Fig. 1 is a transmission line or something like that, which has a complex impedance Z rather than the pure resistance R. The extension of the present theory to such a general case is straightforward — all we have to do is to modify the last term of Eq. (3). We would then observe, for example, that the extra shift would exhibit an oscillatory behavior which is superposed on the exponential decay. However, the main conclusions do

not change because, for example, the irrelevance of the battery in the energy consumption processes relies on the fact that F_0 does not appear explicitly in Eqs. (2) and (3) — this fact remains true when we modify the last term of Eq. (3). For this reason, we believe that the present paper has revealed bare essentials of the electrical-pulse generation by the virtual photo-excitation.

To summarize, we have considered electrical-pulse generation in the "virtual excitation" regime. The electronic system is any electro-optic material including a quantum well structure (QWS) biased by a dc electric field, which is applied to induce large $\chi^{(2)}$. The energy transfer is analyzed when the electronic system is coupled to an external circuit. It is found that the photon *frequency* is subject to an extra red shift in addition to the usual self-phase modulation, whereas the photon *number* is conserved. The extra red shift approaches zero in both limits of zero and infinite impedance of the circuit. It is also shown that an external battery, which produces the dc bias field in the QWS, supplies *no* net energy, and the Joule energy consumed in the external circuit is supplied only from the extra red shift of the optical field.

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[8] Neither the multi-level effect nor the exciton-exciton interaction alter the main conclusions of the present paper.
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[11] The third-order effects, for example, modify the decay constant C_0R in Eqs. (5)-(11). This, however, does not alter the main conclusions such as $U_R = U_{ERS}$ and $U_{V_0} = 0$.
[12] We include in R the internal resistance of the external battery.
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FIG. 1. A schematic diagram of the system under consideration. An optical pulse goes into a capacitor, the center region of which is made of a biased QWS or another EO material. An electrical pulse is generated in the center region, and the current J flows in the external circuit. We find that the optical pulse, after it passes through the capacitor, is subject to an extra red shift, in addition to the usual self phase modulation which occurs in the initial and final transients.

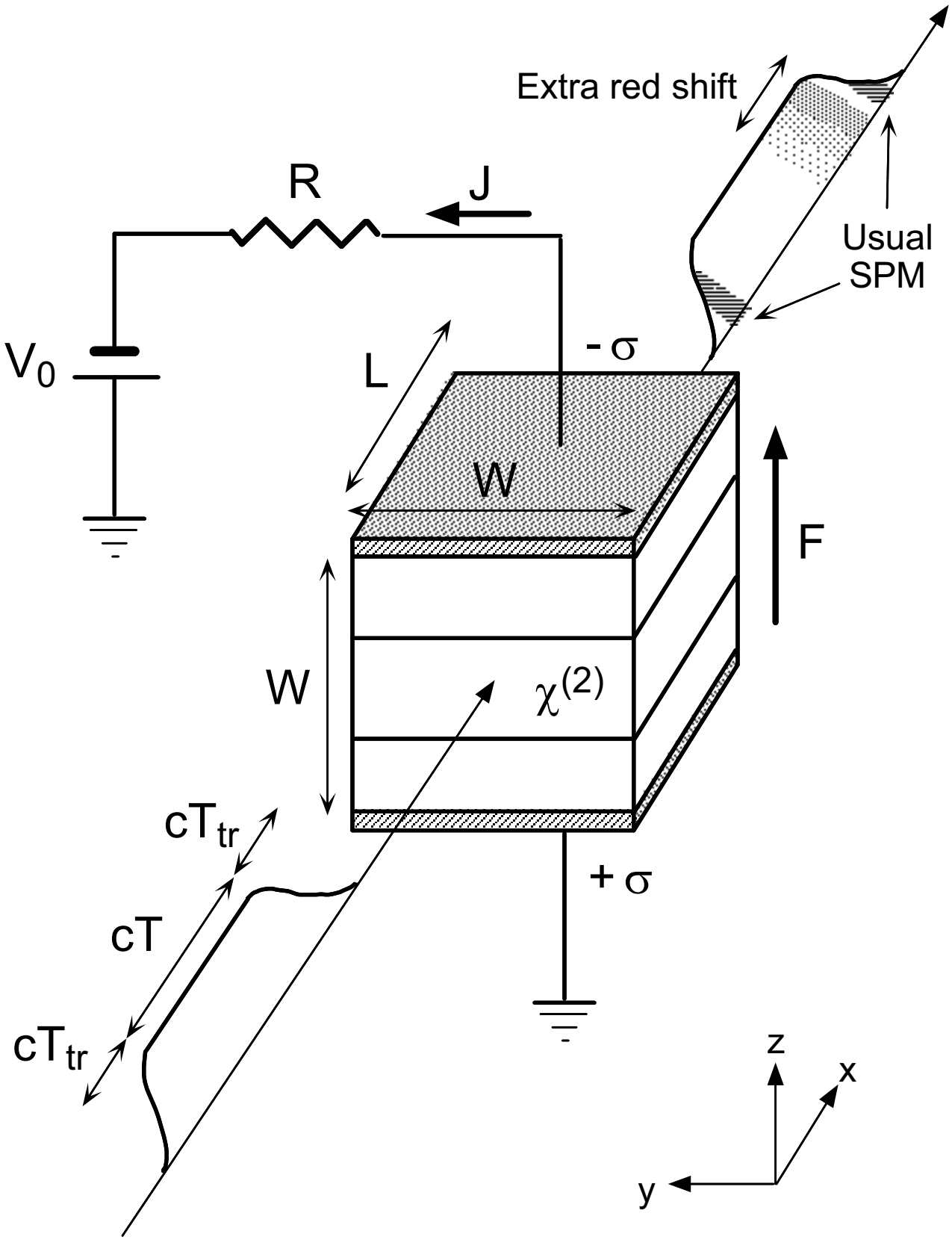

Fig. 1 A. Shimizu and M. Yamanishi